Learning to Learn

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23-11-2017
Learning to Learn

Introduction

- No universally agreed upon definition.
- Possible definitions:
  - Replace hand-crafted components of learning algorithms with learned ones.
  - Have an outer learning algorithm that seeks to optimise an inner algorithm.
- e.g.
  - Learning to optimise.
  - Few-shot learning.
- Synonymous with Meta-Learning.
(Old & well-known) idea: View learning as an optimisation problem.

\[ \theta^* = \arg\min_{\theta} f(\theta) \]

e.g.
Learning to Optimise
Learning as Optimisation

➤ (Old & well-known) idea: View learning as an optimisation problem.

➤

\[ \theta^* = \arg \min_{\theta} f(\theta) \]

e.g.

➤ MLE:

\[ f(\theta) := -\log p(D | \theta) \]
Learning to Optimise

Learning as Optimisation

▶ (Old & well-known) idea: View learning as an optimisation problem.

\[ \theta^* = \arg\min_{\theta} f(\theta) \]

e.g.

▶ MLE: \[ f(\theta) := - \log p(D | \theta) \]
▶ VB: \[ f(\theta) := KL[q_\theta(\cdot) \parallel p(\cdot | D)] \]
(Old & well-known) idea: Pick a function \( g \) where

\[
\theta_{t+1} = g(f, \theta_{1:t})
\]

Construct \( g \) s.t. \( \theta_t \to \theta^* \) as \( t \) becomes large. (Hopefully)

e.g.
(Old & well-known) idea: Pick a function $g$ where

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Construct $g$ s.t. $\theta_t \rightarrow \theta^*$ as $t$ becomes large. (Hopefully)

e.g.

(S)GD-like: $g(f, \theta_{1:t}) := \theta_t - \eta(f, \theta_{1:t}) \nabla f(\theta_t)$
Learning to Optimise

Iterative Optimisation

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▶ Bayes Opt.: $g(f, \theta_{1:t}) = \arg\min_{\theta} \mathcal{A}(\theta, p(\hat{f} | \theta_{1:t}, y_{1:t}))$
Learning to Optimise

Iterative Optimisation

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▶ Construct $g$ s.t. $\theta_t \to \theta^*$ as $t$ becomes large. (Hopefully)

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▶ Bayes Opt.: $g(f, \theta_{1:t}) = \text{argmin}_\theta \mathcal{A}(\theta, p(\hat{f} | \theta_{1:t}, y_{1:t})$)

▶ Observation: $g$ is hand-crafted (up to a couple of free parameters).
Learning to Optimise

Learning to Iteratively Optimise

- (New-ish) idea: Learn \( g \).
- e.g. Parameterise \( g \)

\[
\theta_{t+1} = g_\varphi (f, \theta_{1:t})
\]

and learn \( \varphi \) from data. (data = optimisation problems)
Learn a controller for the step-size of SGD [Daniel et al., 2016]:

\[
g_\phi (f, \theta_{1:t}) = \theta_t - \exp (\phi^T \hat{\phi}_t) \nabla f (\theta_t), \quad \hat{\phi}_t = \phi (\theta_t, \hat{\phi}_{t-1}),
\]

where \( \phi \) is a hand-crafted vector-valued basis function.

**Advantages**
- Low memory footprint
- Small number of parameters

**Disadvantages**
- Hand-engineered features
- No second-order info. used
Learning to Optimise
“Learning to Learn by Gradient Descent by Gradient Descent” [Andrychowicz et al., 2016]

\[ g_\varphi (f, \theta_{1:t}) = \theta_t - r_\varphi (h_t), \quad h_t = H_\varphi (h_{t-1}, \nabla f (\theta_{t-1})) \]

Advantages
- Flexible
- Variable dimensionality

Disadvantages
- Large memory footprint
- No coupling
Learning to Optimise
“Learning to Optimize” [Li and Malik, 2016]

Learn an MLP autoregressor $m_\varphi$:

$$g_\varphi (f, \theta_{1:t}) = m_\varphi (\theta_t, f (\theta_{t-H:t}), \nabla f (\theta_{t-H:t})),$$

for some $H \in \mathbb{N}$.

Advantages
- Flexible
- Coupling between variables

Disadvantages
- Large memory footprint
- Not flexible in no. dims.
Learning to Optimise

“Learning to Learn without Gradient Descent by Gradient Descent” [Chen et al., 2017]

Let $g_\phi$ be an RNN $r_\phi$ with hidden state $h_t$ given by

$$g_\phi (f, \theta_{1:t}) = r_\phi (h_t), \quad h_t = H_\phi (h_{t-1}, \theta_{t-1}, f (\theta_{t-1})).$$

Advantages

- Flexible parametrisation
- Coupling between dims

Disadvantages

- Large memory footprint
- Requires $\nabla f$ during training
- Fixed dimensionality.
Learning to Optimise
“Learning to Learn without Gradient Descent by Gradient Descent” [Chen et al., 2017]

- EI training criterion:

\[
L_{EI}(\theta) = -\mathbb{E}_{f, y_{1:T-1}} \left[ \sum_{t=1}^{T} EI(\theta_t | y_{1:t-1}) \right]
\]

Computed via GP
Learning to Optimise

“Learning to Learn without Gradient Descent by Gradient Descent” [Chen et al., 2017]

Figure 3. Average minimum observed function value, with 95% confidence intervals, as a function of search steps on functions sampled from the training GP distribution. Left four figures: Comparing DNC with different reward functions against Spearmint with fixed and estimated GP hyper-parameters, TPE and SMAC. Right bottom: Comparing different DNCs and LSTMs. As the dimension of the search space increases, the DNC’s performance improves relative to the baselines.
Learning to optimise is conceptually appealing.

Empirical results seem promising.

Memory requirements large relative to simple methods.
  
  Probably a careful trade-off required between flexibility and overhead.
Learning to Learn
Data-efficient learning

- Problem: ‘Insufficient’ training data
- Use related data
- k-shot learning
  - Image classification
  - Sentence completion
Learning to Learn

k-shot learning

Large $\tilde{\mathcal{D}} = \{\tilde{u}_i, \tilde{y}_i\}_{i=1}^{\tilde{N}}$, $\tilde{y}_i \in \{1, \ldots, \tilde{C}\}$

Small $\mathcal{D} = \{u_i, y_i\}_{i=1}^{kC}$, $y_i \in \{\tilde{C} + 1, \ldots, \tilde{C} + C\}$
Learning to Learn

k-shot learning

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Small $D = \{u_i, y_i\}_{i=1}^{kC}, y_i \in \{\tilde{C} + 1, \ldots, \tilde{C} + C\}$

- LSTM meta-learner
- Learning on learnt features
Learning to Learn
“Optimisation as a model for few-shot learning” [Ravi and Larochelle, 2017]

\[ \theta_t = \theta_{t-1} - \alpha_t \nabla \theta_{t-1} L_t \]

\[ c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t \]

- Learn \( f_t, i_t, c_0 \)
Learning to Learn

Learning on features

Figure 1: Learning on features [Burgess et al., 2016]

- Train a Neural Network classifier on $\tilde{D}$
  - Train on embedded features obtained from last hidden layer
- Common baseline: Nearest neighbour matching
Learning to Learn

“Matching Networks for One Shot Learning” [Vinyals et al., 2016]

\[ a(\hat{x}, x_i) = \frac{e^{c(f(\hat{x},S),g(x_i,S))}}{\sum_{j=1}^{k} e^{c(f(\hat{x},S),g(x_j,S))}} \]

\[ \hat{y} = \sum_{i=1}^{k} a(\hat{x}, x_i)y_i \]
Learning to Learn

“Discriminative k-shot learning using probabilistic models” [Bauer et al., 2017]

- Bayesian approach on softmax weights
- Found single Gaussian worked best

\[
p(W|\mathcal{D}, \tilde{\mathcal{D}}) \propto \mathcal{N}(W|\mu_{MAP}, \Sigma_{MAP}) \prod_{n=1}^{N} p(y_n|x_n, W)
\]
Learning to Learn

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<table>
<thead>
<tr>
<th>Method</th>
<th>1-shot</th>
<th>5-shot</th>
</tr>
</thead>
<tbody>
<tr>
<td>ResNet-34 + Isotropic Gaussian (ours)</td>
<td>56.3 ± 0.4%</td>
<td>73.9 ± 0.3%</td>
</tr>
<tr>
<td>Matching Networks (reimplemented, 1-shot)</td>
<td>46.8 ± 0.5%</td>
<td>-</td>
</tr>
<tr>
<td>Matching Networks (reimplemented, 5-shot)</td>
<td>-</td>
<td>62.7 ± 0.5%</td>
</tr>
<tr>
<td>Meta-Learner LSTM (Ravi &amp; Larochelle, 2017)</td>
<td>43.4 ± 0.8%</td>
<td>60.6 ± 0.7%</td>
</tr>
<tr>
<td>Prototypical Nets (1-shot) (Snell et al., 2017)</td>
<td>49.4 ± 0.8%</td>
<td>65.4 ± 0.7%</td>
</tr>
<tr>
<td>Prototypical Nets (5-shot) (Snell et al., 2017)</td>
<td>45.1 ± 0.8%</td>
<td>68.2 ± 0.7%</td>
</tr>
</tbody>
</table>
Learning to Learn
“Few-Shot Image Recognition by Predicting Parameters from Activations”
[Qiao et al., 2017]

\[ P(y_i|x_i) = \frac{e^{a(x_i) \cdot \phi(E_s[s_{y_i}])}}{Z} \]

- Statistic set (1st moment)
- At k-shot train, each sample is new category
Learning to Learn

Potential future research

- Similarities between classes (e.g., animals)
  - Attempted with GMMs [Bauer et al., 2017]
- Combine feature-learning with learning on features for general k-shot
  - Currently ‘fine-tuning’ is heuristic
Learning to learn by gradient descent by gradient descent.

Discriminative k-shot learning using probabilistic models.
arXiv:1706.00326 [cs, stat].

One-Shot Learning in Discriminative Neural Networks.
NIPS Bayesian Deep Learning Workshop.

Learning to learn without gradient descent by gradient descent.
In International Conference on Machine Learning, pages 748–756.

Learning step size controllers for robust neural network training.
In AAAI, pages 1519–1525.

Li, K. and Malik, J. (2016).
Learning to optimize.

Few-Shot Image Recognition by Predicting Parameters from Activations.
arXiv:1706.03466 [cs].
Optimization as a model for few-shot learning.

Matching Networks for One Shot Learning.
*Advances in Neural Information Processing Systems.*